Effective Thinking Through Mathematics

Modern Mathematics: Plan II (M310P), Spring 2021

Instructor: Professor Michael Starbird, starbird@mail.utexas.edu

Office hours: MWF 10:00-11:00.

Teaching Assistant: Casandra Monroe, cmonroe@utexas.edu

Office hours and Workshop Session hours: To be determined

Texts: *The Heart of Mathematics: An invitation to effective thinking, 4th edition* by Edward B. Burger and Michael Starbird, Wiley & Sons, 2012.

The 5 Elements of Effective Thinking, by Edward B. Burger and Michael Starbird, Princeton University Press, 2012.

Homework Exercises:

Homework will be due regularly, usually each Sunday at midnight. Work will be submitted via Canvas. Clarity of exposition is extremely important. Thus, you should strive for well-written, polished answers. Collaboration on these assignments with others from *this class* is encouraged after you have worked on all the questions on your own. Submitted work must be individually written up and all collaborators should be acknowledged. One part of some of the assignments will ask you to take a mathematical concept and create a story illustrating how that idea could have been conceived and developed.

Workshop Sessions:

Once every two weeks you will meet with Casandra with a small group of about five students to discuss the mathematics. You will sign up for these sessions during the first couple weeks of class. Sessions will be held for 30 minutes each. Your performance in the workshop sessions will be partially measured by your presentation and discussion of selected homework challenges with your peers and Casandra.

Tell the Story of Concept Creation Assignments:

Every idea in mathematics is a natural result of a process by which new ideas are created. Throughout the semester, you will be asked to describe why various mathematical ideas were created. We want you to have the feeling, “I could have thought of that.” Specifics of these assignments will be provided.

Creative Work Illuminating Mathematics Assignment:

You will be asked to produce a creative work that illuminates one of the mathematical ideas we encounter. Your work might be a musical composition, a work of visual art, a play, or some other creative work. It will include an essay that describes how the mathematical idea is being celebrated in the work. The work will be graded on three dimensions: the accuracy and depth of the mathematics, the creativity of the work, and the quality of the execution. Details of this project will be given later.

Quality of Failure Assignment:

Learning from missteps and mistakes is one of the most valuable methods of making forward progress. This assignment asks you to write a brief statement of how you used an initial failure to help you come to deeper understanding or to a good creative idea in this course.

Math as Metaphor Project:

As a final project, choose an issue that you care about outside mathematics. Then produce a paper or work that illustrates how you created a new insight or product in that other field by using the elements of effective thinking that we discuss during the semester. This project will be described more fully during the semester.

Grading Policy:

The course goal is to have you learn some great mathematical ideas and use the thinking lessons they teach in areas of your life beyond mathematics courses.

Homework exercises	30%
Workshop sessions	10%
Tell the Story of Concept Creation	30%
Creative Work Illuminating Mathematics	10%
Quality of Failure	5%
Math as Metaphor	15%

Attendance and participation are required. Roughly, every three unexcused absences will reduce the final grade by one step.

Philosophy of the course:

Most people do not have an accurate picture of mathematics. For many, mathematics is the torture of tests, homework, and problems, problems, problems. The very word *problems* suggests unpleasantness and anxiety. But mathematics is not "problems."

Some people view mathematics as a set of formulas to be applied to a list of problems at the ends of textbook chapters. Toss that idea into the trash. Formulas in algebra, trigonometry, and calculus are incredibly useful. But, in this course, you will see that mathematics is a network of intriguing ideas—not a dry, formal list of techniques. It is creative, powerful, and even artistic.

Mathematics uses penetrating techniques of thought that we can all use to solve problems, analyze situations, and sharpen the way we look at our world. This course emphasizes basic strategies of thought and analysis. These strategies have their greatest value to us in dealing with real-life decisions and situations that are completely outside mathematics. These "life lessons," inspired by mathematical thinking, empower us to better grapple with and conquer the problems and issues that we all face in our lives from love to business, from art to politics. If you can conquer infinity and the fourth dimension, then what can't you do?

The realm of mathematics contains some of the greatest ideas of humankind—ideas comparable to the works of Shakespeare, Plato, and Michelangelo. These mathematical ideas helped shape history, and they can add texture, beauty, and wonder to our lives.

The road through this course is not free from perils, bumps, and jolts. Sometimes you will confront issues that start beyond your comprehension. The journey to true understanding can be difficult and frustrating, but stay the course and be patient. There is light at the end of the tunnel—and throughout the journey, too.

What's the point of it all? Well, the bottom line is that mathematics involves profound ideas. Making these ideas our own empowers us with the strength, the techniques, and the confidence to accomplish wonders. And, hopefully, you will find the ideas and the process of thinking through them to be an enjoyable, as well as valuable, experience.

Topics: The mathematical topics that we will consider will be selected from those listed below. The titles are taken from the textbook *The Heart of Mathematics: An invitation to effective thinking, 4th Edition*, which was written for this course. The dates on which the topics are considered depend on how the class proceeds.

Fun and Games: An introduction to rigorous thought

Silly Stories All Having a Moral: *Conundrums that evoke techniques of effective thinking*

Number Contemplation

Counting: *How the Pigeonhole principle leads to precision through estimation*

Numerical Patterns in Nature: *Discovering nature's beauty and the Fibonacci numbers*

Prime Cuts of Numbers: *How the prime numbers are the building blocks of all natural numbers*

Secret Codes and How to Become a Spy: *How modular arithmetic and primes lead to secret public codes*

The Irrational Side of Numbers: *Are there numbers beyond fractions?*

Get Real: *The point of decimals and pinpointing numbers on the real line*

Infinity

Beyond Numbers: *What does infinity mean?*

Comparing the Infinite: *Pairing up collections via a one-to-one correspondence*

The Missing Member: *Are some infinities larger than others?*

Geometric Gems

Pythagoras and His Hypotenuse: *How a puzzle leads to the proof of one of the gems of mathematics*

A View of an Art Gallery: *Using computational geometry to place security cameras in museums*

The Sexiest Rectangle: *Finding aesthetics in life, art, and math through the Golden Rectangle*

The Platonic Solids Turn Amorous: *Discovering the symmetry and interconnections among the Platonic Solids*

The Shape of Reality?: *How straight lines can bend in non-Euclidean geometries*

The Fourth Dimension: *Can you see it?*

Contortions of Space

Rubber Sheet Geometry: *Discovering the topological idea of equivalence by distortion*

The Band That Wouldn't Stop Playing: *Experimenting with the Möbius Band and Klein Bottle*

Modeling through Graphs

Circuit Training: *From the Königsberg Bridge Conundrum to graphs*

Feeling Edgy?: *Exploring relationships among vertices, edges, and faces*

Chaos and Fractals

Images: *Viewing a gallery of fractals*

The Dynamics of Change: *Can change be modeled by repeated applications of simple processes?*

The Infinitely Detailed Beauty of Fractals: *How to create works of infinite intricacy through repeated processes*

Predetermined Chaos: *How repeated simple processes result in utter chaos*

Between Dimensions: *Can the dimensions of fractals fall through the cracks?*

Taming Uncertainty

Chance Surprises: *Some scenarios involving chance that confound our intuition*

Predicting the Future in an Uncertain World: *How to measure uncertainty through the idea of probability*

Random Thoughts: *Are coincidences as truly amazing as they first appear?*

Drizzling, Defending, and Doctoring: *Probability in our world and our lives*

Meaning from Data

Stumbling Through a Minefield of Data: *Inspiring statistical concepts through pitfalls*

Getting Your Data to Shape Up: *Organizing, describing, and summarizing data*

Looking at Super Models: *Mathematically described distributions*

Go Figure: *Making inferences from data*

War, Sports, and Tigers: *Statistics throughout our lives*

Deciding Wisely

Great Expectations: *Weighing the unknown future through the notion of expected value*

Peril at the Polls: *Deciding who actually wins an election*

Cutting Cake for Greedy People: *Envy-free allocation*